## Schedule and Abstracts

# Workshop

# 'Singularity formation and long-time behavior in dispersive PDEs'

March 14 – 18, 2016 in Bonn

Organizers:

Roland Donninger
Herbert Koch

**Monday, March 14, 2016**

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|---|---|
| 08:30-09:00 | *Registration* |
| 09:00-09:40 | Piotr Bizoń: Wave maps on a wormhole |
| 09:50-10:30 | Patrick Gérard: Long time analysis of the half wave equation |
| 10:30-11:00 | *Coffee break* |
| 11:00-11:40 | Sung-Jin Oh: On energy critical geometric wave equations |
| 11:50-12:30 | Andrew Lawrie: Wave maps on hyperbolic space |
| 12:30-14:00 | *Lunch break* |
| 14:00-14:40 | Benjamin Dodson: Global well-posedness results for some cubic nonlinear equations |
| 14:50-15:30 | Marius Beceanu: Reversed Strichartz inequalities and their applications |
| 15:30-16:00 | *Coffee break* |
| 16:00-16:40 | Sigmund Selberg:Lower bounds on the radius of spatial analyticity for DKG and KdV equations |
| 16:50-17:30 | Jason Metcalfe: Local energy decay for wave equations on nontrapping asymptotically flat space-times |

**Tuesday, March 15, 2016**

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| 09:00-09:40 | Nikolay Tzvetkov: Quasi-invariant gaussian measures for the 4th order NLS under sharp regularity assumptions |
| 09:50-10:30 | Kenji Nakanishi: Global dynamics of the nonlinear Schrodinger equation with a potential |
| 10:30-11:00 | *Coffee break* |
| 11:00-11:40 | Piero D'Ancona: Scattering in the energy space for the NLS with variable coefficients |
| 11:50-12:30 | Tadahiro Oh: On the well-posedness of the cubic fourth order NLS on the circle |
| 12:30-14:00 | *Lunch break* |
| 14:00-14:40 | Jean-Claude Saut: Long time existence for some water waves system |
| 14:50-15:30 | Tetsu Mizumachi: Stability of line solitons for the KP-II equation |
| 15:30-16:00 | *Coffee break* |
| 16:00-16:40 | Alexandru Ionescu: Global solutions of the gravity-capillary water wave system in 3 dimensions |
| 16:50-17:30 | Daniel Ioan Tataru: Long time dynamics for water waves |
| 17:40-18:05 | Benjamin Harrop-Griffiths: Finite depth gravity water waves in holomorphic coordinates |

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| 18:05-18:30 | Dana Mendelson: Symplectic non-squeezing for Hamiltonian PDEs. |

**Wednesday, March 16, 2016**

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| 09:00-09:40 | Luis Vega Gonzalez: An isoperimetric-type inequality for Dirac Hamiltonians with electrostatic shell interactions. |
| 09:50-10:30 | Vedran Sohinger: A rigorous derivation of the defocusing cubic nonlinear Schrodinger equation on 2D and 3D rectangular tori |
| 10:30-11:00 | *Coffee break* |
| 11:00-11:40 | Junfeng Li: The Global well-posedness of 2D Benjamin problem in energy spaces |
| 11:50-12:30 | Jeremy Marzuola: Traveling waves for a model of gravity-driven film flows in cylindrical domains |
| 12:30-14:00 | *Lunch break* |
| 14:00-14:40 | Claudio Muñoz: Asymptotic stability in nonlinear scalar field equations |
| 14:50-15:30 | Rowan Killip: Conservation laws for integrable PDE |
| 15:30-16:00 | *Coffee break* |
| 16:00-16:40 | Michael Struwe: Scattering for a critical nonlinear wave equation in 2 space dimensions |
| 16:50-17.15 | Jonas Luehrmann: Concentration Compactness for the Critical Maxwell-Klein-Gordon Equation |
| 19:30 | *Conference Dinner* |

**Thursday, March 17, 2016**

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| 09:00-09:40 | Raphaël Côte: Around the soliton resolution for wave type equations |
| 09:50-10:30 | Frank Merle: Soliton resolution along a sequence of times for the focusing energy critical wave equation |
| 10:30-11:00 | *Coffee break* |
| 11:00-11:40 | Hatem Zaag: Existence and stability of a solution with a new prescribed behavior for a heat equation with a critical nonlinear gradient term |
| 11:50-12:15 | Timothy Candy: t.b.a. |
| 12:15-14:00 | *Lunch break* |
| 14:00-14:40 | Jeremie Szeftel: Global regularity for the 2+1 dimensional equivariant Einstein-Wave Map system |
| 14:50-15:30 | Sohrab Shahshahani: On the motion of the free boundary of a self-gravitating incompressible fluid |
| 15:30-16:00 | *Coffee break* |
| 16:00-16:40 | Nicolas Burq: Long time dynamics for the damped Klein Gordon equation |
| 16:50-17:30 | Fabrice Planchon: Reflections on domains and waves propagation |
| 17:40-18:30 | Dirk Hundertmark: t.b.a. |

**Friday, March 18, 2016**

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| 09:00-09:40 | Svetlana Roudenko: Blow-up and scattering in the focusing nonlinear Klein-Gordon equation. |
| 09:50-10:30 | Birgit Schörkhuber: Stable self-similar blowup in supercritical evolution equations |
| 10:30-11:00 | *Coffee break* |
| 11:00-11:40 | Enno Lenzmann: Blowup for biharmonic and fractional NLS |
| 11:50-12:30 | Yvan Martel: Strongly interacting blow up bubbles for the mass critical NLS |
| **12:30** | **End of Workshop** |

# Abstracts

**Marius Beceanu:**
**Reversed Strichartz in-equalities and their applications**

In this talk I will present reversed Strichartz in-equalities and some of their applications.

**Piotr Bizoń:**
**Wave maps on a wormhole**

We consider equivariant wave maps from a static asymptotically flat spacetime with two asymptotic ends into the three-sphere. This is a toy-model for gaining insight into the dissipation-by-dispersion phenomena, in particular the soliton resolution conjecture. Using the hyperboloidal formulation of the initial value problem, we show that, for a given topological degree of the map, all solutions starting from smooth initial data converge to the unique stationary solution as time goes to infinity. The asymptotics of this relaxation process is described in detail. This is joint work with Michal Kahl.

**Nicolas Burq:**
**Long time dynamics for the damped Klein Gordon equation**

For general nonlinear Klein-Gordon equations with dissipation we show that any finite energy radial solution either blows up in finite time or asymptotically approaches a stationary solution in $H^1\times L^2$. In particular, any global in positive times solution is bounded in positive times. The result is very robust and applies to standard energy subcritical focusing nonlinearities $|u|^{p-1} u$, $1<p<(d+2)/(d-2)$ as well as to any energy subcritical nonlinearity obeying a sign condition of the Ambrosetti-Rabinowitz type. The argument involves both techniques from nonlinear dispersive PDEs and dynamical systems (invariant manifold theory in Banach spaces and convergence theorems). This is a joint work with W. Schlag (U. Chicago) and G. Raugel (Université Paris-Sud).

**Timothy Candy:**
**t.b.a.**

**Raphaël Côte:**
**Around the soliton resolution for wave type equations**

I will talk about a few recent results for equivariant wave maps and the energy critical wave equations, around the soliton resolution. It roughly states that for large time, the solutions decompose into stationnary solutions living at different scales, and a regular part. I will sketch the links between the proofs for each equation.

**Piero D' Ancona:**
**Scattering in the energy space for the NLS with variable coefficients**

In a joint work with B.Cassano (Roma), we prove global existence and scattering in the energy space for the NLS with variable coefficients in dimension 3 or larger. The metric is assumed to be a small long range perturbation of the flat metric, plus lower order terms and a potential with large positive part. The nonlinearity is defocusing and energy subcritical. The crucial tool is a bilinear smoothing (interaction Morawetz) estimate which we can prove also in the case of a star-shaped exterior domain with Dirichlet boundary conditions. In addition, we prove Strichartz estimates for the variable coefficient equation perturbed by a potential with a large positive part.

**Benjamin Dodson:**
**Global well–posedness results for some cubic nonlinear equations**

In this talk we will discuss global well-posedness theory for the cubic wave and Schrodinger equations in three dimensions. In particular we will discuss several type one and type two blowup results.

**Patrick Gérard:**
**Long time analysis of the half wave equation**

The one dimensional half wave equation with cubic nonlinearity is a simple example of a non dispersive nonlinear wave evolution. In this talk, I will discuss the state of the art of the problem of long time growth of high Sobolev norms for this equation.

**Benjamin Harrop-Griffiths:**
**Finite depth gravity water waves in holomorphic coordinates**

We consider irrotational gravity water waves with finite bottom in holomorphic coordinates. We will discuss the local well-posedness of the problem in this context and establish cubic lifespan bounds for solutions with small initial data. This is joint work with Mihaela Ifrim and Daniel Tataru.

**Dirk Hundertmark:**
**t.b.a.**

**Alexandru Ionescu:**
**Global solutions of the gravity-capillary water wave system in 3 dimensions**

I will discuss some recent work, joint with Yu Deng, Benoit Pausader, and Fabio Pusateri, on the global existence of solutions of the full gravity-capillary water wave system in 3D. The main new issues in this problem are the slow pointwise decay of solutions and the presence of a large set of resonances.

**Rowan Killip:**
**Conservation laws for integrable PDE**

I will describe a one-parameter family of exact conservation laws for the Korteweg--de Vries equation. These permit us to obtain global uniform bounds on solutions for a wide range of

Besov norms. In particular, we cover the cases $H^s$ for $-1\leq s < 1$. Our method treats the circle and the line cases on equal footing and applies also to other integrable systems such as (complex) mKdV and NLS. This is joint work with M Visan and X Zhang.

**Andrew Lawrie:**
**Wave maps on hyperbolic space**

The wave maps problem on hyperbolic space exhibits several interesting features of a different nature than the corresponding problem on flat space. We'll investigate several of these features in this talk, including the global asymptotic stability of a continuous one-paramater family of equivariant harmonic maps in 2 space dimensions indexed by how far the map wraps around the target. As a first step towards examining the stability of these stationary wave maps with respect to non-equivariant perturbations we'll prove a small data global well-posedness and scattering result for wave maps on hyperbolic space in high dimensions, while developing some of the analytic tools needed to treat the 2d problem. The proof, which uses the moving frame approach introduced by Shatah and Struwe, relies crucially on the fact that the main dynamic equations in Tao's caloric gauge are scalar, rather than the tensorial equations that arise in say, the Coulomb gauge. This talk is based on joint work with Sung-Jin Oh and Sohrab Shahshahani.

**Enno Lenzmann:**
**Blowup for biharmonic and fractional NLS**

I will discuss recent results concerning the existence of blowup solutions and universal upper bounds on blowup rates for biharmonic and fractional NLS. The talk is based on joint work with T. Boulenger and D. Himmelsbach.

**Junfeng Li:**
**The Global well-posedness of 2D Benjamin problem in energy spaces**

In this talk, I will present our recent work on the well-posedness of 2D Benjamin problem. It is an KP-I problem with an Benjamin term in the equation. I will present our argument on the bilinear estimates one the resonant set are more complicated than KP-I problem.

**Jonas Luehrmann:**
**Concentration Compactness for the Critical Maxwell-Klein-Gordon Equation**

We discuss a proof of global regularity, scattering and a priori bounds for solutions to the energy critical Maxwell-Klein-Gordon equation relative to the Coulomb gauge for essentially arbitrary smooth data of finite energy. The proof is based upon a novel "twisted" Bahouri-Gérard type profile decomposition and a concentration compactness/rigidity argument by Kenig-Merle, following the method developed by Krieger-Schlag in the context of critical wave maps. This is joint work with Joachim Krieger.

**Yvan Martel:**
**Strongly interacting blow up bubbles for the mass critical NLS**

This is a work in collaboration with Pierre Raphael (University of Nice). We construct a new class of multi-solitary wave solutions for the mass critical two dimensional nonlinear Schrodinger equation (NLS). Given any integer K>1, there exists a global solution of (NLS) that decomposes asymptotically into a sum of solitary waves centered at the vertices of a K-sided regular polygon and concentrating at a logarithmic rate in large time. This solution blows up in infinite time with logarithmic rate. Using the pseudo-conformal transform, this yields the first example of solution blowing up in finite time with a rate strictly above the pseudo-conformal one. Such solution concentrates K bubbles at a point. These special behaviors are due to strong interactions between the waves, in contrast with previous works on multi-solitary waves of (NLS) where interactions do not affect the blow up rate.

**Jeremy Marzuola:**
**Traveling waves for a model of gravity-driven film flows in cylindrical domains**

With Roberto Camassa, H. Reed Ogrosky and N. Vaughn, we study traveling wave solutions for a recently-derived model of a falling viscous film on the interior of a vertical rigid tube. By identifying a Hopf bifurcation and using numerical continuation, families of non-trivial traveling wave solutions may be traced out in parameter space. These families all contain a single solution at a `turnaround point' with larger film thickness than all others in the family. In an earlier paper, it was conjectured that this turnaround point may represent a critical thickness separating two distinct flow regimes observed in physical experiments as well as two distinct types of behavior in transient solutions to the model. Here, these hypotheses are verified over a range of parameter values using a combination of numerical and analytical techniques. The linear stability of these solutions is also discussed; both large- and small-amplitude solutions are shown to be unstable, though the instability mechanisms are different for each wave type. Specically, for small-amplitude waves, the region of relatively flat film away from the localized wave crest is subject to the same instability that makes the trivial flat-film solution unstable; for large amplitude waves, this mechanism is present but dwarfed by a much stronger tendency to relax to a regime close to that followed by small-amplitude waves.

**Dana Mendelson:**
**Symplectic non-squeezing for Hamiltonian PDEs**

In this talk, I will discuss symplectic non-squeezing for the nonlinear Klein-Gordon equation (NLKG) which can be (formally) regarded as an infinite dimensional Hamiltonian system. The symplectic phase space for this equation is at the critical regularity, and in this setting there is no global well-posedness nor any uniform control on the local time of existence for arbitrary initial data. We will present several non-squeezing results for the NLKG, including a conditional result which states that uniform bounds on the Strichartz norms of solutions for initial data in bounded subsets of the phase space imply global-in-time non-squeezing. The proofs rely on several approximation results for the flow, which we obtain using a combination of probabilistic and deterministic techniques.

### Frank Merle:
### Soliton resolution along a sequence of times for the focusing energy critical wave equation

We consider the soliton resolution conjecture for global solutions to the focusing energy critical wave equation, in space dimension 3,4 or 5. We prove it along a sequence of times. This is an important step towards the full soliton resolution in the nonradial case and without any size restrictions.

### Jason Metcalfe:
### Local energy decay for wave equations on nontrapping asymptotically flat space-times

This is a joint work with J. Sterbenz and D. Tataru on local energy decay for the wave equation. In the stationary case, we show that trapping and eigenvalues / resonances are the only obstructions to local energy decay. Moreover, we show that these results are stable for time-dependent perturbations.

### Tetsu Mizumachi:
### Stability of line solitons for the KP-II equation

The KP-II equation was derived by Kadmotsev and Petviashvili to explain stability of line solitary waves of shallow water. In this talk, I will prove stability of $1$-line solitons for perturbations in $(1+x^2)^{-1/2-0}H^1(\mathbb{R}_{x,y}^2)$.

**Claudio Muñoz:**
**Asymptotic stability in nonlinear scalar field equations**

The purpose of this talk is to describe how the mechanism of asymptotic stability works for solitons in some scalar field equations in dimension one, only using data in the energy space, and the knowledge of a modified Fermi golden rule. We will also discuss some possible extensions to other dispersive models. This is joint work with Michal Kowalczyk and Yvan Martel.

**Kenji Nakanishi:**
**Global dynamics of the nonlinear Schrodinger equation with a potential**

Nonlinear dispersive equations are known to have various types of solutions, including scattering, blow-up and solitons. It is also important to distinguish stable and unstable solitons, since the former are asymptotic profiles of generic solutions, while the latter appear on the thresholds between different types of solutions. The goal of this study is to describe the global Hamiltonian dynamics in a phase space which contains all of those solutions. It aims not only for the final asymptotic states, but also for intermediate regimes, especially when the solution is changing its behavior. We take the nonlinear Schrodinger equation with a potential as a simple model case, assuming that the linear part has a unique and negative eigenvalue. Restricting the phase space with radial symmetry, small mass, and an energy upper bound slightly above the soliton of the second least energy, we can classify the behavior of solutions into 9 cases, where the blow-up and the scattering to the ground states in positive time are separated by the codimension-1 centerstable manifold of the first excited states, and those in negative time are separated by the center-unstable manifold.

**Sung-Jin Oh:**
**On energy critical geometric wave equations**

I will survey and present recent results concerning the question of global regularity and scattering for geometric wave equations in energy critical dimensions. Prime examples of such equations include the wave map, Max-well-Klein-Gordon and Yang-Mills equations.

**Tadahiro Oh:**
**On the well-posedness of the cubic fourth order NLS on the circle**

We consider the Cauchy problem for the cubic fourth order NLS (4NLS) on the circle in low regularities. In particular, we show that the 4NLS is ill-posed below L^2. Then, we consider well-posedness of the renormalized 4NLS in both deterministic and probabilistic setting. This talk is based on joint work with Nikolay Tzvetkov (Universite de Cergy-Pontoise) and Yuzhao Wang (University of Edinburgh).

**Fabrice Planchon:**
**Reflections on domains and waves propagation**

We will review recent results on how linear waves propagate on domains with boundaries, with emphasis on constructing sufficientely good approximations to quantify dispersive effects.

**Svetlana Roudenko:**
**Blow-up and scattering in the focusing nonlinear Klein-Gordon equation**

We discuss the focusing nonlinear Klein-Gordon equation starting with the cubic nonlinearity in 3 dimensions. Inspired by the paper of Donninger-Schlag on the same equation, we further investigate the blow up and scattering behavior of its solutions. We extend the theoretical boundaries of the blow up regions and discuss the behavior of solutions there, as well as we study the properties of the scattering tails. Some of the methods are applicable for other dimensions and nonlinearities. Parts of this work is joint with T. Duyckaerts and also with K. Yang and Y. Zhao.

**Jean-Claude Saut:**
**Long time existence for some water waves system**

Asymptotic models for water waves are derived in different regimes when one or several relevant parameters are small. The full rigorous justification of those models requires in particular to establish the well-posedness of the Cauchy problem on "long" time scales, of the order of inverse of the small parameters (together with suitable bounds). We will present recent long time existence results, in particular for Boussinesq type systems.

**Birgit Schörkhuber:**
**Stable self-similar blowup in supercritical evolution equations**

In recent years a canonical method has been developed to study the stability of self-similar blowup solutions in energy supercritical semilinear wave equations, with applications to the focusing NLW, Wave maps and the Yang-Mills equation in supercritical dimensions. However, blowup via self-similar solutions is not a particular feature of dispersive equations but is observed also in various parabolic models. In this talk I will give an outline of the approach and discuss the generalisation of our techniques to supercritical heat flows. This is joint work with Roland Donninger.

**Sigmund Selberg:**
**Lower bounds on the radius of spatial analyticity for DKG and KdV equations**

We consider the Cauchy problem with initial data that are real analytic with a uniform radius of analyticity, and study the persistence of spatial real analyticity for later times. Explicit estimates on the radius of analyticity are obtained, using a modification of the Fourier restriction norm method. This has implications for the convergence rate of certain numerical schemes.

**Sohrab Shahshahani:**
**On the motion of the free boundary of a self-gravitating incompressible fluid**

The motion of the boundary of an incompressible fluid body subject to its self-gravitatioal force can be described by the free boundary problem of the Euler-Poisson system. This problem differs from the water was problem in that the constant gravity in water waves is replaced by a nonlinear self-gravitaty. In this talk, we present some recent results on the well-posedness of this problem and give a lower bound on the lifespan of smooth solutions. In particular, we show that the Taylor sign condition always holds leading to local well-posedness, and for smooth data of size $\epsilon$ a unique smooth solution exists for time greater than or equal to $O(1/{\epsilon}^2)$. This is achieved by constructing an appropriate quantity and a coordinate transformation such that the new quantity in the new coordinate system satisfies an equation without quadratic nonlinearities. This is joint work with L. Bieri, S. Miao, and S. Wu.

**Vedran Sohinger:**
**A rigorous derivation of the defocusing cubic nonlinear Schrodinger equation on 2D and 3D rectangular tori**

We study the defocusing cubic nonlinear Schrodinger equation on 2D and 3D rectangular tori. The problem that we consider is the rigorous derivation of this equation from the dynamics of many-body quantum systems.

In particular, we study the problem in the context of the limit of the BBGKY hierarchy, which gives rise to the Gross-Pitaevskii (GP) hierarchy. An important point in the analysis is to show uniqueness of solutions of the GP hierarchy. This problem was fully settled on the 2D classical torus by Kirkpatrick, Schlein, and Staffilani. In their work, the authors use the combinatorial boardgame argument due to Klainerman and Machedon. Furthermore, they apply spacetime estimates for the free Schrodinger evolution associated to density matrices.

In the first part of the talk, I will study the problem on the 3D classical torus. The limit of the BBGKY hierarchy in this context was shown to be the GP hierarchy by Elgart, Erdos, Schlein, and Yau. However, the uniqueness step remained open. I will show how to obtain uniqueness by using the weak quantum de Finetti theorem, which was first used in this context on R^3 by Chen, Hainzl, Pavlovic, and Seiringer.

In the second part of the talk, I will show how to extend these ideas to general rectangular tori. In particular, one can consider irrational tori. This is joint work with Sebastian Herr. There are challenges in studying general rectangular tori due to the fact that the free Schrodinger evolution is no longer necessarily periodic in time. In our work, we obtain an alternative approach to prove the relevant spacetime estimate for the free Schrodinger evolution. Consequently, we are able to obtain a sharp range of regularity exponents which are the same as in the case of the classical torus.

**Michael Struwe:**
**Scattering for a critical non-linear wave equation in 2 space dimensions**

In joint work with Martin Sack we show that the solutions to the Cauchy problem for a wave equation with critical exponential nonlinearity in 2 space dimensions scatter for arbitrary smooth, compactly supported initial data.

**Jeremie Szeftel:**
**Global regularity for the 2+1 dimensional equivariant Einstein-Wave Map system**

We consider the equivariant 2+1 dimensional Einstein-wave map system and show that if the target satisfies the so called Grillakis condition, then global existence holds. In view of the fact that the 3+1 vacuum Einstein equations with a spacelike translational Killing field reduce to a 2+1 dimensional Einstein-wave map system with target the hyperbolic plane, which in particular satisfies the Grillakis condition, this work proves global existence for the equivariant class of such space times. This is a joint work with L. Andersson and N. Gudapati.

**Daniel Tataru:**
**Long time dynamics for water waves**

The aim of this talk is to survey a series of recent results concerning lifespan bounds for water waves in several interesting physical scenarios.

**Nikolay Tzvetkov:**
**Quasi-invariant gaussian measures for the 4th order NLS under sharp regularity assumptions**

We will present a very recent result showing that the gaussians measures formally defined as $Z^{-1}\exp(-\|u\|_{H^s}^2)du$ are quasi-invariant under the flow of the 4th order cubic NLS, provided $s>1/2$. The assumption on $s$ is sharp in the sense that such a result cannot hold for $s<1/2$. We will also present a result showing that the situation drastically change in a lack of dispersion. This is a joint work with Tadahiro Oh and Philippe Sosoe.

**Luis Vega Gonzales:**

**An isoperimetric-type inequality for Dirac Hamiltonians with electrostatic shell interactions.**

Institutions: UPV and BCAM, Bilbao, Spain.

I shall present some recent work in collaboration with N. Arrizabalaga and A. Mas about the spectral properties of the coupling H+aV, where H is the massive free Dirac operator in 3d, and aV is an electrostatic shell potential (which depends on the coupling real parameter a) located on the boundary of a smooth domain. Our main result is an isoperimetric-type inequality for the admissible range of a's for which the coupling H+aV generates pure point spectrum in (−m,m). That the ball is the unique optimizer of this inequality is also shown. Regarding some ingredients of the proof, we make use of the Birman-Schwinger principle adapted to our setting in order to prove first some monotonicity property of the admissible a's, and then a sharp constant of a quadratic form inequality, from which the isoperimetric-type inequality is derived.

**Hatem Zaag:**
**Existence and stability of a solution with a new prescribed behavior for a heat equation with a critical nonlinear gradient term**

We consider a nonlinear heat equation with a double source: a power of the solution and a power of its gradient, in the critical case. We construct a solution blowing up in finite time only at the origin, and give its profile. To do so, we linearize the similarity variables' version around the profile, and show that the negative part of the spectrum can be controlled thanks to the bounding effect of the linear operator. We are left only with the positive directions of the solution, in finite number. They are controlled near zero thanks to the use of index theory. The interpretation of the parameters of the finite-dimensional problems in terms of the blow-up time and the blow-up point give us the stability of the constructed solution with respect to initial data. Remarkably, our solution shows a new type of behavior, unknown for the semilinear heat equation (with power nonlinearity).